Homework V (i)

1. From the definition $m^*(A)$, show that

$$m^*(A) = \inf\{m(G) : \text{open } G \supseteq A\}$$

$$= \inf\{m(G) : \text{open } G \text{ & } G_0 \supseteq G \supseteq A\},$$

whenever G_0 is an open set containing A.

2. Let $E \subseteq G$. Show that, $\forall\, U \subseteq \mathbb{R}$,

$$E \setminus U = E \setminus (U \cap G)$$

and

$$m^*\left(E \triangle (U \cap G)\right) \leq m^*\left(E \triangle U\right)$$

3. Let $m \ni E \subseteq (a,b) \subseteq \mathbb{R}$ and $\varepsilon > 0$. Show that $\exists$ disjoint open intervals $I_1, I_2, \cdots I_n$ contained in (a,b) s.t.

$$m\left(E \triangle \bigcup_{i=1}^{n} I_i\right) < \varepsilon,$$

in two methods:

(a) Using Q1

(b) Using Q2 (and (i)⇒(iv) of the 1st principle of Littlewood for $m^*(E)<+\infty$)